Vectors

GCSE Mathematics • **Higher Tier** • Grades 8–9 • Non-calculator

WORKED SOLUTIONS

Name: worked example Class: _____ Total: 62 / 62

Show all your working. Marks are given for correct method as well as the final answer.

1. Work out $\begin{pmatrix}3\\2\end{pmatrix} + \begin{pmatrix}1\\4\end{pmatrix}$. **[2 marks]**

add the tops, add the bottoms

Answer: $\begin{pmatrix}4\\6\end{pmatrix}$

2. Work out $\begin{pmatrix}5\\1\end{pmatrix} - \begin{pmatrix}2\\3\end{pmatrix}$. **[2 marks]**

5 − 2 = 3
1 − 3 = −2

Answer: $\begin{pmatrix}3\\-2\end{pmatrix}$

3. Work out $3\begin{pmatrix}2\\-1\end{pmatrix}$. **[2 marks]**

multiply both parts by 3

Answer: $\begin{pmatrix}6\\-3\end{pmatrix}$

4. Work out $2\begin{pmatrix}1\\3\end{pmatrix} + \begin{pmatrix}4\\-2\end{pmatrix}$. **[3 marks]**

$2\begin{pmatrix}1\\3\end{pmatrix} = \begin{pmatrix}2\\6\end{pmatrix}$
$\begin{pmatrix}2\\6\end{pmatrix} + \begin{pmatrix}4\\-2\end{pmatrix} = \begin{pmatrix}6\\4\end{pmatrix}$

Answer: $\begin{pmatrix}6\\4\end{pmatrix}$

5. Work out the magnitude of $\begin{pmatrix}3\\4\end{pmatrix}$. **[3 marks]**

$\sqrt{3^2 + 4^2}$
$= \sqrt{25} = 5$
it is just Pythagoras

Answer: 5

6. Work out the magnitude of $\begin{pmatrix}-5\\12\end{pmatrix}$. **[3 marks]**

25 + 144 = 169
the minus disappears when squared

Answer: 13

7. $\overrightarrow{AB} = \begin{pmatrix}4\\-3\end{pmatrix}$. Write down $\overrightarrow{BA}$. **[2 marks]**

BA = −AB
reverse both signs

Answer: $\begin{pmatrix}-4\\3\end{pmatrix}$

8. $\mathbf{a} = \begin{pmatrix}2\\1\end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix}-1\\3\end{pmatrix}$. Work out $\mathbf{a} + 2\mathbf{b}$. **[3 marks]**

$2\mathbf{b} = \begin{pmatrix}-2\\6\end{pmatrix}$
$\mathbf{a} + 2\mathbf{b} = \begin{pmatrix}0\\7\end{pmatrix}$

Answer: $\begin{pmatrix}0\\7\end{pmatrix}$

9. Using the same **a** and **b**, work out $3\mathbf{a} - \mathbf{b}$. **[3 marks]**

$3a = \begin{pmatrix}6\\3\end{pmatrix}$
$-b = \begin{pmatrix}1\\-3\end{pmatrix}$
sum $= \begin{pmatrix}7\\0\end{pmatrix}$

Answer: $\begin{pmatrix}7\\0\end{pmatrix}$

10. Show that $\begin{pmatrix}2\\3\end{pmatrix}$ and $\begin{pmatrix}6\\9\end{pmatrix}$ are parallel. **[3 marks]**

$\begin{pmatrix}6\\9\end{pmatrix} = 3\begin{pmatrix}2\\3\end{pmatrix}$
parallel = one is a multiple of the other

Answer: the second is 3 × the first

11. Explain whether $\begin{pmatrix}2\\5\end{pmatrix}$ and $\begin{pmatrix}4\\9\end{pmatrix}$ are parallel. **[3 marks]**

$2 \times 2 = 4$ ✓
but $2 \times 5 = 10$, not 9
not a single multiple, so not parallel

Answer: no

12. $\begin{pmatrix}3\\k\end{pmatrix}$ is parallel to $\begin{pmatrix}9\\15\end{pmatrix}$. Work out k. **[3 marks]**

$9 \div 3 = 3$, so the factor is 3
$15 \div 3 = 5$

Answer: $k = 5$

13. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. Write $\overrightarrow{AB}$ in terms of **a** and **b**. **[3 marks]**

AB = AO + OB
$= -a + b$
go BACK along a, then along b

Answer: $b - a$

14. Using the same diagram, write $\overrightarrow{BA}$ in terms of **a** and **b**. **[2 marks]**

the reverse of AB

Answer: $a - b$

15. M is the midpoint of AB, where $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. Write $\overrightarrow{AM}$. **[3 marks]**

AB = b − a
M is halfway, so $AM = \frac{1}{2}AB$

Answer: $\frac{1}{2}(b - a)$

16. With the same M, write $\overrightarrow{OM}$ in terms of **a** and **b**. **[4 marks]**

OM = OA + AM
$= a + \frac{1}{2}(b - a)$
$= \frac{1}{2}a + \frac{1}{2}b$

Answer: $\frac{1}{2}(a + b)$

17. P divides AB in the ratio $1 : 2$. Write $\overrightarrow{AP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. **[4 marks]**

1 : 2 means P is $\frac{1}{3}$ of the way

$AP = \frac{1}{3}(b - a)$

Answer: $\frac{1}{3}(b - a)$

18. A point moves by $\begin{pmatrix}3\\1\end{pmatrix}$ then by $\begin{pmatrix}-1\\4\end{pmatrix}$. Write down the single vector for the whole journey. **[2 marks]**

add them

Answer: $\begin{pmatrix}2\\5\end{pmatrix}$

19. $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, and C is such that $\overrightarrow{OC} = 3\mathbf{a}$.

(a) Write $\overrightarrow{BC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. **[3 marks]**

$BC = BO + OC$

$= -b + 3a$

$= 3a - b$

(b) Write $\overrightarrow{AB}$, and hence say whether BC is parallel to AB. **[3 marks]**

$AB = b - a$

$3a - b$ is not a multiple of $b - a$

so NOT parallel

Answer: $3a - b$; not parallel

20. $\mathbf{p} = \begin{pmatrix}4\\-2\end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix}-1\\5\end{pmatrix}$.

(a) Work out $2\mathbf{p} + 3\mathbf{q}$. **[3 marks]**

$2p = \begin{pmatrix}8\\-4\end{pmatrix}$

$3q = \begin{pmatrix}-3\\15\end{pmatrix}$

sum $= \begin{pmatrix}5\\11\end{pmatrix}$

(b) Work out the magnitude of $\mathbf{p}$, in surd form. **[3 marks]**

$\sqrt{16 + 4} = \sqrt{20}$

$= 2\sqrt{5}$

Answer: $\begin{pmatrix}5\\11\end{pmatrix}$; $2\sqrt{5}$

Free to print and use at home or in class. Please do not re-host or sell. Written by Hampstead Tuition Centre from the published exam specifications.