Sine and Cosine Rule

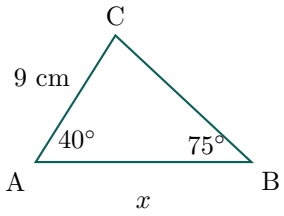
GCSE Mathematics • **Higher Tier** • Grades 7–8 • Non-calculator

WORKED SOLUTIONS

Name: worked example Class: _____ Total: 74 / 74

Show all your working. Marks are given for correct method as well as the final answer.

1. Work out the length x, to 1 d.p. **[3 marks]**



two angles and a side → SINE rule

angle C = 180 − 40 − 75 = 65°

pair each side with the angle OPPOSITE it

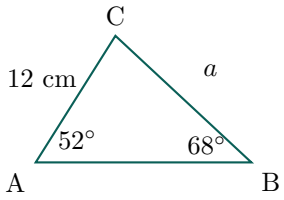
$\frac{x}{\sin 65} = \frac{9}{\sin 75}$

$x = 9 \times \sin 65 \div \sin 75$

$= 8.446... = 8.4$ cm

Answer: 8.4 **cm**

2. Work out the length a, to 1 d.p. **[3 marks]**



sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B}$

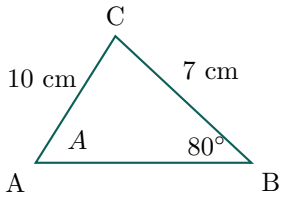
$\frac{a}{\sin 52} = \frac{12}{\sin 68}$

$a = 12 \times \sin 52 \div \sin 68$

$= 10.198... = 10.2$ cm

Answer: 10.2 **cm**

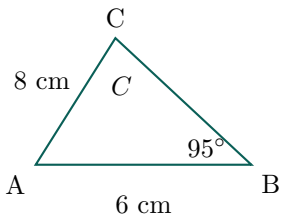
3. Work out the angle A, to 1 d.p. **[3 marks]**



$\frac{\sin A}{7} = \frac{\sin 80}{10}$
$\sin A = 7 \times \sin 80 \div 10$
$\sin A = 0.68937...$
$A = 43.57... = 43.6°$

Answer: 43.6 °

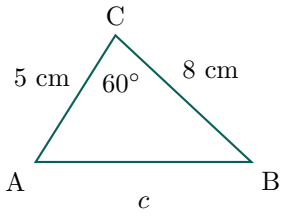
4. Work out the angle C, to 1 d.p. **[3 marks]**



$\frac{\sin C}{6} = \frac{\sin 95}{8}$
$\sin C = 6 \times \sin 95 \div 8 = 0.74714...$
$C = 48.35... = 48.3°$

Answer: 48.3 °

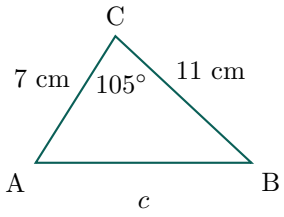
5. Work out the length c, to 1 d.p. **[4 marks]**



two sides and the angle BETWEEN → COSINE rule
$c^2 = a^2 + b^2 - 2ab \cos C$
$= 64 + 25 - 2(8)(5)\cos 60$
$= 89 - 80(0.5) = 49$
$c = 7.0$ cm

Answer: 7.0 **cm**

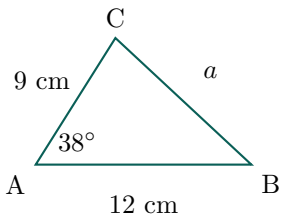
6. Work out the length c, to 1 d.p. **[4 marks]**



$c^2 = 121 + 49 - 2(11)(7)\cos 105$
$\cos 105 = -0.2588$
a negative cosine makes the side LONGER
$= 170 + 39.86 = 209.86$
$c = 14.486 = 14.5$ cm

Answer: 14.5 **cm**

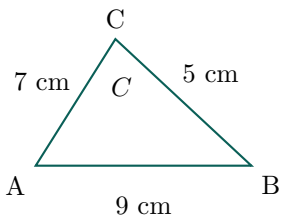
7. Work out the length a, to 1 d.p. **[4 marks]**



$a^2 = 81 + 144 - 2(9)(12)\cos 38$
$= 225 - 216(0.78801)$
$= 225 - 170.21 = 54.79$
$a = 7.402... = 7.4$ cm

Answer: 7.4 **cm**

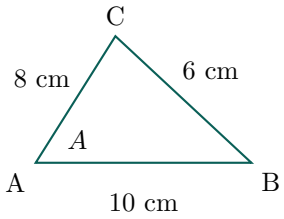
8. Work out the angle C, to 1 d.p. **[4 marks]**



three sides → COSINE rule rearranged
$\cos C = \frac{a^2+b^2-c^2}{2ab}$
$= \frac{25+49-81}{2(5)(7)} = \frac{-7}{70} = -0.1$
negative cosine means an OBTUSE angle
$C = 95.7°$

Answer: 95.7 °

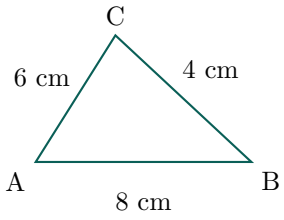
9. Work out the angle A, to 1 d.p. **[4 marks]**



$\cos A = \frac{b^2+c^2-a^2}{2bc}$
$= \frac{64+100-36}{2(8)(10)} = \frac{128}{160} = 0.8$
$A = 36.86... = 36.9°$

Answer: 36.9 °

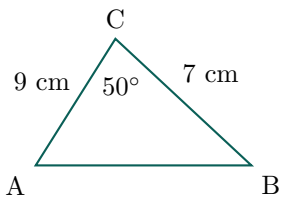
10. Work out the largest angle in a triangle with sides 4 cm, 6 cm and 8 cm, to 1 d.p. **[4 marks]**



the largest angle faces the LONGEST side
$\cos C = \frac{16+36-64}{2(4)(6)}$
$= \frac{-12}{48} = -0.25$
$C = 104.5°$

Answer: 104.5 °

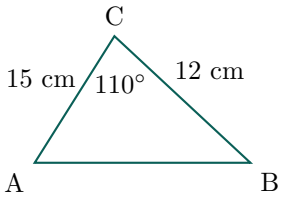
11. Work out the area, to 1 d.p. **[3 marks]**



area $= \frac{1}{2}ab \sin C$
$= \frac{1}{2}(7)(9)\sin 50$
$= 31.5 \times 0.766$
$= 24.13 = 24.1 \text{ cm}^2$

Answer: 24.1 cm^2

12. Work out the area, to 1 d.p. **[3 marks]**



$\frac{1}{2}(12)(15)\sin 110$
$= 90 \times 0.9397$
$= 84.57 = 84.6 \text{ cm}^2$

Answer: 84.6 **cm²**

13. A triangle has area 30 cm², two sides of 10 cm and 8 cm. Work out the angle between them, to 1 d.p. **[4 marks]**

$30 = \frac{1}{2}(10)(8)\sin C$
$30 = 40 \sin C$
$\sin C = 0.75$
$C = 48.59 = 48.6°$

Answer: 48.6 °

14. Write down which rule you would use when you know three sides and want an angle. **[1 mark]**

no angle-side PAIR is known, so sine will not start

Answer: the cosine rule

15. Write down which rule you would use when you know two angles and one side. **[1 mark]**

an angle and its opposite side make a pair

Answer: the sine rule

16. Explain why the sine rule can give two possible angles. **[2 marks]**

sin 30 = sin 150
the second angle is 180 – the first

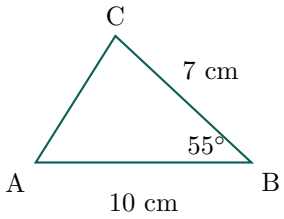
Answer: sin is positive for acute and obtuse angles

17. $\sin A = 0.6$ and A is obtuse. Work out A, to 1 d.p. **[3 marks]**

the calculator gives 36.87°
obtuse, so 180 – 36.87
= 143.1°

Answer: 143.1 °

18. A triangle has $AB = 10$ cm, $BC = 7$ cm and angle $ABC = 55°$.



(a) Work out AC, to 1 d.p. **[4 marks]**

cosine rule (two sides, angle between)

$b^2 = 100 + 49 - 2(10)(7)\cos 55$

$= 149 - 140(0.5736) = 68.7$

$AC = 8.288... = 8.3$ cm

(b) Work out the area of the triangle, to 1 d.p. **[3 marks]**

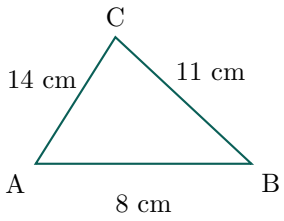
$\frac{1}{2}(10)(7)\sin 55$

$= 35 \times 0.8192$

$= 28.67 = 28.7$ cm^2

Answer: 8.3 cm ; 28.7 cm^2

19. In triangle PQR, $PQ = 8$ cm, $QR = 11$ cm and $PR = 14$ cm.



(a) Work out the largest angle, to 1 d.p. **[4 marks]**

largest angle faces 14 cm, so it is angle Q

$\cos Q = \frac{64+121-196}{2(8)(11)}$

$= \frac{-11}{176} = -0.0625$

$Q = 93.58 = 93.6°$

(b) Work out the area, to 1 d.p. **[3 marks]**

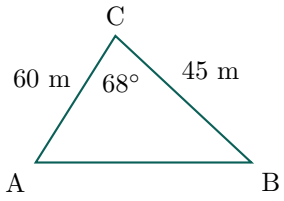
$\frac{1}{2}(8)(11)\sin 93.58$

$= 44 \times 0.99805$

$= 43.9$ cm^2

Answer: 93.6° ; 43.9 cm^2

20. A field is a triangle with two sides 60 m and 45 m and an angle of 68° between them.



(a) Work out the area, to the nearest m^2. **[3 marks]**

$\frac{1}{2}(60)(45)\sin 68$

$= 1350 \times 0.92718$

$= 1251.7 = 1252\ m^2$

(b) Work out the length of the third side, to 1 d.p. **[4 marks]**

$c^2 = 3600 + 2025 - 2(60)(45)\cos 68$

$= 5625 - 5400(0.37461)$

$= 5625 - 2022.9 = 3602.1$

$c = 60.0\ m$

Answer: ____ $1252\ m^2$; $60.0\ m$____

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