Sampling and Capture-Recapture

GCSE Mathematics • **Higher Tier** • Grades 5–7 • Non-calculator

WORKED SOLUTIONS

Name: worked example Class: _____ Total: 56 / 56

Show all your working. Marks are given for correct method as well as the final answer.

1. 40 fish are caught, marked and returned. Later 50 are caught and 8 are marked. Estimate the population. **[3 marks]**

$\frac{8}{50}$ of the lake is marked
but only 40 fish were marked in total
$N = \frac{40 \times 50}{8}$
$= 250$

Answer: 250

2. 60 birds are ringed. A later sample of 80 contains 12 ringed. Estimate the population. **[3 marks]**

$N = \frac{60 \times 80}{12}$
$= 400$

Answer: 400

3. 25 deer are tagged. A sample of 30 contains 5 tagged. Estimate the population. **[3 marks]**

$N = \frac{25 \times 30}{5}$
$= 150$

Answer: 150

4. 120 beetles are marked. A sample of 45 contains 9 marked. Estimate the population. **[3 marks]**

$N = \frac{120 \times 45}{9}$
$= 600$

Answer: 600

5. Write down the capture-recapture formula. **[2 marks]**

the proportion marked in the sample
equals the proportion marked in the population

Answer: $N = \frac{\text{first sample} \times \text{second sample}}{\text{marked in second}}$

6. Write down one assumption the capture-recapture method relies on. **[1 mark]**

no births, deaths or migration between the samples

Answer: the population does not change

7. Write down a second assumption. **[1 mark]**

otherwise the second sample is not random

Answer: the marked animals mix back in fully

8. Write down a third assumption about the marks themselves. **[1 mark]**

a lost tag makes a marked animal count as unmarked

Answer: no marks are lost

9. Explain what happens to the estimate if some marks fall off. **[3 marks]**

fewer marked are counted in the second sample
a smaller denominator makes N larger
so the estimate is an OVERestimate

Answer: ____ it becomes too big ____

10. Explain what happens to the estimate if marked animals are easier to catch. **[3 marks]**

more marked appear in the second sample
a bigger denominator makes N smaller

Answer: ____ it becomes too small ____

11. Explain what a random sample means. **[2 marks]**

no group is favoured

Answer: ____ every member has an equal chance of being chosen ____

12. A survey about sport is carried out at a gym. Explain why the sample is biased. **[2 marks]**

the sample is not representative of everyone

Answer: ____ gym users are more sporty than average ____

13. A school has 900 pupils. A stratified sample of 60 is taken. Work out the sampling fraction. **[2 marks]**

$\frac{60}{900} = \frac{1}{15}$

Answer: _____ $\frac{1}{15}$ _____

14. In that school, Year 11 has 180 pupils. Work out how many are in the stratified sample. **[3 marks]**

$180 \times \frac{1}{15}$
$= 12$

Answer: _____ 12 _____

15. In that school, Year 7 has 240 pupils. Work out how many are in the sample. **[2 marks]**

$240 \div 15 = 16$

Answer: _____ 16 _____

16. Explain why a stratified sample is better than a simple random one when the groups differ in size. **[2 marks]**

a random sample might miss a small group entirely

Answer: ____ every group is represented in proportion ____

17. In a sample of 80 people, 18 said yes. Estimate how many of 2 000 would say yes. **[3 marks]**

$\frac{18}{80} = 0.225$
$0.225 \times 2000 = 450$

Answer: _____ 450 _____

18. 200 trout are tagged in a lake. A later sample of 150 contains 15 tagged.

(a) Estimate the number of trout in the lake. **[3 marks]**

$N = \frac{200 \times 150}{15}$

$= 2000$

(b) Explain why the estimate might be unreliable. **[3 marks]**

the population may have changed

tags may have been lost

any ONE clear assumption earns the mark

Answer: 2000 ; assumptions may not hold

19. A college has 1 200 students: 500 in Year 12 and 700 in Year 13. A stratified sample of 96 is taken.

(a) Work out how many Year 12 students are sampled. **[3 marks]**

fraction $= \frac{96}{1200} = 0.08$

500×0.08

$= 40$

(b) Work out how many Year 13 students are sampled. **[2 marks]**

$700 \times 0.08 = 56$

check: $40 + 56 = 96$ ✓

Answer: 40 ; 56

20. 90 rabbits are marked and released. Two weeks later 60 are caught, of which 4 are marked.

(a) Estimate the rabbit population. **[3 marks]**

$N = \frac{90 \times 60}{4}$

$= 1350$

(b) Between the two catches, several marked rabbits died. Explain the effect on the estimate. **[3 marks]**

fewer marked rabbits are left to catch

so the second sample has fewer marked

the denominator drops and N rises

so the estimate is too big

Answer: 1350 ; an overestimate

Free to print and use at home or in class. Please do not re-host or sell. Written by Hampstead Tuition Centre from the published exam specifications.