Recurring Decimals and Rational Numbers

GCSE Mathematics • **Higher Tier** • Grades 7–8 • Non-calculator

WORKED SOLUTIONS

Name: worked example Class: _____ Total: 61 / 61

Show all your working. Marks are given for correct method as well as the final answer.

1. Write $0.\dot{3}$ as a fraction. **[2 marks]**

$x = 0.333...$
$10x = 3.333...$
$9x = 3$
$x = \frac{3}{9} = \frac{1}{3}$

Answer: $\frac{1}{3}$

2. Write $0.\dot{7}$ as a fraction. **[2 marks]**

$x = 0.777...$
$10x = 7.777...$
$9x = 7$

Answer: $\frac{7}{9}$

3. Write $0.\dot{4}\dot{5}$ as a fraction in its simplest form. **[3 marks]**

two digits repeat, so use 100x
$x = 0.4545...$, $100x = 45.4545...$
$99x = 45$
$\frac{45}{99} = \frac{5}{11}$

Answer: $\frac{5}{11}$

4. Write $0.\dot{2}\dot{7}$ as a fraction in its simplest form. **[3 marks]**

$100x - x = 27.2727... - 0.2727...$
$99x = 27$
$\frac{27}{99} = \frac{3}{11}$

Answer: $\frac{3}{11}$

5. Write $0.\dot{1}2\dot{3}$ as a fraction in its simplest form. **[3 marks]**

three digits repeat, so use 1000x
$999x = 123$
$\frac{123}{999} = \frac{41}{333}$

Answer: $\frac{41}{333}$

6. Write $0.1\dot{6}$ as a fraction in its simplest form. **[4 marks]**

only the 6 repeats, so shift TWICE
$10x = 1.666...$, $100x = 16.666...$
subtract the two that end the same
$90x = 15$
$\frac{15}{90} = \frac{1}{6}$

Answer: $\frac{1}{6}$

7. Write $0.2\dot{7}$ as a fraction in its simplest form. **[4 marks]**

$10x = 2.777..., \ 100x = 27.777...$
$90x = 25$
$\frac{25}{90} = \frac{5}{18}$

Answer: $\frac{5}{18}$

8. Write $0.4\dot{1}\dot{6}$ as a fraction in its simplest form. **[4 marks]**

$10x = 4.1616..., \ 1000x = 416.1616...$
$990x = 412$
$\frac{412}{990} = \frac{206}{495}$

Answer: $\frac{206}{495}$

9. Write $1.\dot{6}$ as a mixed number in its simplest form. **[3 marks]**

the whole number stays outside
$0.666... = \frac{6}{9} = \frac{2}{3}$

Answer: $1\frac{2}{3}$

10. Write $2.\dot{3}\dot{6}$ as a mixed number in its simplest form. **[4 marks]**

$0.3636... : 99x = 36$
$\frac{36}{99} = \frac{4}{11}$

Answer: $2\frac{4}{11}$

11. Write $\frac{5}{9}$ as a recurring decimal. **[2 marks]**

$5 \div 9 = 0.5555...$

Answer: $0.\dot{5}$

12. Write $\frac{4}{11}$ as a recurring decimal. **[2 marks]**

$4 \div 11 = 0.363636...$

Answer: $0.\dot{3}\dot{6}$

13. Write $\frac{7}{12}$ as a decimal. **[2 marks]**

$7 \div 12 = 0.58333...$

Answer: $0.58\dot{3}$

14. Explain how you can tell from its denominator that $\frac{3}{8}$ terminates. **[3 marks]**

a fraction terminates when the denominator's only prime factors are 2 and 5
because $10 = 2 \times 5$

Answer: $8 = 2^3$, only prime factor 2

15. Explain why $\frac{5}{6}$ recurs. **[2 marks]**

$6 = 2 \times 3$
the 3 forces it to recur

Answer: 6 has a factor of 3

16. Write down whether $\frac{9}{40}$ terminates or recurs, with a reason. **[2 marks]**

$40 = 2^3 \times 5$
only 2s and 5s

Answer: terminates

17. Explain why every recurring decimal is a rational number. **[2 marks]**

the algebraic method always produces $\frac{a}{b}$
with a and b whole numbers

Answer: it can be written as a fraction

18. Write down whether $\sqrt{9}$ is rational, with a reason. **[2 marks]**

$\sqrt{9} = 3 = \frac{3}{1}$
a surd that simplifies to an integer is rational

Answer: rational

19. Prove that $0.\dot{9} = 1$.

(a) Use the algebraic method. **[3 marks]**

$x = 0.999...$
$10x = 9.999...$
$9x = 9$
$x = 1$

(b) Write down what this shows about decimal representations. **[2 marks]**

one number can have two decimal forms
0.999... and 1 are the same number

Answer: $x = 1$; a number can have two decimal forms

20. $x = 0.\dot{2}\dot{4}$ and $y = 0.\dot{1}\dot{2}$.

(a) Write x and y as fractions in their simplest form. **[4 marks]**

$99x = 24 \rightarrow \frac{24}{99} = \frac{8}{33}$
$99y = 12 \rightarrow \frac{12}{99} = \frac{4}{33}$

(b) Work out $x - y$ as a fraction in its simplest form. **[3 marks]**

$\frac{8}{33} - \frac{4}{33}$
$= \frac{4}{33}$

Answer: $\frac{8}{33}$ and $\frac{4}{33}$; $\frac{4}{33}$

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