Quadratic and Other Sequences

GCSE Mathematics • **Higher Tier** • Grades 6–8 • Non-calculator

WORKED SOLUTIONS

Name: worked example Class: _____ Total: 68 / 68

Show all your working. Marks are given for correct method as well as the final answer.

1. Write down the next two terms of 2, 6, 12, 20, ... **[2 marks]**

differences: 4, 6, 8, ...
next differences 10 and 12
20 + 10 = 30, 30 + 12 = 42

Answer: 30, 42

2. Work out the second difference of 3, 8, 15, 24, 35. **[2 marks]**

first differences: 5, 7, 9, 11
second differences: 2, 2, 2

Answer: 2

3. Explain how the second difference tells you the n^2 coefficient. **[2 marks]**

second difference 2 → $1n^2$
always HALVE it

Answer: it is half the second difference

4. Work out the nth term of 3, 8, 15, 24, 35. **[4 marks]**

second difference 2, so start with n^2
n^2: 1, 4, 9, 16, 25
subtract: 2, 4, 6, 8, 10
that is 2n
nth term = $n^2 + 2n$

Answer: $n^2 + 2n$

5. Work out the nth term of 4, 7, 12, 19, 28. **[4 marks]**

first differences 3, 5, 7, 9
second difference 2 → n^2
subtract n^2: 3, 3, 3, 3, 3

Answer: $n^2 + 3$

6. Work out the nth term of 2, 8, 18, 32, 50. **[4 marks]**

first differences 6, 10, 14, 18
second difference 4, halve it → $2n^2$
$2n^2$: 2, 8, 18, 32, 50 ✓

Answer: $2n^2$

7. Work out the nth term of 0, 3, 8, 15, 24. **[4 marks]**

second difference 2 → n^2
subtract: −1, −1, −1, −1

Answer: $n^2 - 1$

8. Work out the nth term of 5, 12, 23, 38, 57. **[5 marks]**

first differences 7, 11, 15, 19
second difference 4 $\rightarrow$ $2n^2$
$2n^2$: 2, 8, 18, 32, 50
subtract: 3, 4, 5, 6, 7
that is $n + 2$

Answer: $2n^2 + n + 2$

9. Work out the nth term of 1, 6, 15, 28, 45. **[5 marks]**

first differences 5, 9, 13, 17
second difference 4 $\rightarrow$ $2n^2$
$2n^2$: 2, 8, 18, 32, 50
subtract: $-1, -2, -3, -4, -5$
that is $-n$

Answer: $2n^2 - n$

10. Use the nth term $n^2 + 2n$ to work out the 10th term. **[2 marks]**

$100 + 20$
$= 120$

Answer: 120

11. Show that 63 is a term of the sequence with nth term $n^2 - 1$. **[3 marks]**

$n^2 - 1 = 63$
$n^2 = 64$
$n = 8$
8 is a whole number, so yes

Answer: yes, $n = 8$

12. Show that 50 is NOT a term of the sequence with nth term $n^2 + 2n$. **[3 marks]**

$n^2 + 2n = 50$
$n^2 + 2n - 50 = 0$
$n = 6.14...$ (not a whole number)
the position must be a positive INTEGER

Answer: n is not a whole number

13. Write down the next two terms of 3, 6, 12, 24, ... **[2 marks]**

geometric: each term $\times 2$

Answer: 48, 96

14. Write down the common ratio of 80, 40, 20, 10. **[2 marks]**

$40 \div 80 = \frac{1}{2}$

Answer: $\frac{1}{2}$

15. Work out the nth term of the geometric sequence 2, 6, 18, 54. **[3 marks]**

first term 2, ratio 3
nth term = ar^{n-1}

Answer: $2 \times 3^{n-1}$

16. The first two terms of a Fibonacci-style sequence are 4 and 7. Write down the next three terms. **[2 marks]**

each term is the sum of the two before
4 + 7 = 11, 7 + 11 = 18, 11 + 18 = 29

Answer: 11, 18, 29

17. Write down the nth term of $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$. **[3 marks]**

tops: 1, 2, 3, 4 → n
bottoms: 2, 3, 4, 5 → n + 1

Answer: $\frac{n}{n+1}$

18. A sequence has nth term $2n^2 - 3n$.

(a) Work out the first three terms. **[2 marks]**

n=1: 2 − 3 = −1
n=2: 8 − 6 = 2
n=3: 18 − 9 = 9

(b) Work out the 12th term. **[2 marks]**

2(144) − 36
= 288 − 36 = 252

Answer: −1, 2, 9 ; 252

19. A sequence starts 7, 10, 15, 22, 31.

(a) Work out the nth term. **[4 marks]**

first differences 3, 5, 7, 9
second difference 2 → n^2
subtract n^2: 6, 6, 6, 6, 6
nth term = n^2 + 6

(b) Work out the 20th term. **[2 marks]**

400 + 6
= 406

Answer: n^2 + 6 ; 406

20. A sequence has nth term $n^2 + n$.

(a) Show that every term is even. **[3 marks]**

$n^2 + n = n(n + 1)$

n and $n + 1$ are consecutive

one of any two consecutive numbers is even

so the product is always even

(b) Work out which term equals 156. **[3 marks]**

$n(n + 1) = 156$

$12 \times 13 = 156$

so the 12th term

Answer: ____ $n(n+1)$, one factor is even ; the 12th term____

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