Quadratic Formula and Completing the Square

GCSE Mathematics • **Higher Tier** • Grades 8–9 • Non-calculator

WORKED SOLUTIONS

Name: worked example Class: _____ Total: 61 / 61

Show all your working. Marks are given for correct method as well as the final answer.

1. Write down a, b and c for $2x^2 + 5x - 3 = 0$. **[2 marks]**

c is NEGATIVE 3

Answer: $a = 2,\ b = 5,\ c = -3$

2. Write down a, b and c for $x^2 - 4x + 1 = 0$. **[2 marks]**

b carries its minus sign

Answer: $a = 1,\ b = -4,\ c = 1$

3. Write down the quadratic formula. **[1 mark]**

learn it -- it is not always given

Answer: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

4. Work out the discriminant of $x^2 + 3x + 1 = 0$. **[2 marks]**

$b^2 - 4ac$

$9 - 4 = 5$

Answer: 5

5. Work out the discriminant of $x^2 - 6x + 9 = 0$ and say what it tells you. **[3 marks]**

$36 - 36 = 0$

zero means one repeated solution

Answer: 0, one repeated root

6. Work out the discriminant of $x^2 + x + 4 = 0$ and say what it tells you. **[3 marks]**

$1 - 16 = -15$

negative means no real solutions

Answer: -15, no real roots

7. Solve $x^2 + 5x + 3 = 0$ using the formula, to 2 d.p. **[4 marks]**

$a=1,\ b=5,\ c=3$

disc $= 25 - 12 = 13$

$x = \frac{-5 \pm \sqrt{13}}{2}$

$= -0.697...$ or $-4.302...$

Answer: $x = -0.70$ or -4.30

8. Solve $2x^2 - 3x - 4 = 0$ using the formula, to 2 d.p. **[4 marks]**

$a=2,\ b=-3,\ c=-4$

disc $= 9 + 32 = 41$

$x = \frac{3 \pm \sqrt{41}}{4}$

$= 2.350...$ or $-0.850...$

Answer: $x = 2.35$ or -0.85

9. Solve $x^2 - 4x - 1 = 0$, giving your answers in surd form. **[4 marks]**

disc = 16 + 4 = 20
$x = \frac{4 \pm \sqrt{20}}{2}$
$\sqrt{20} = 2\sqrt{5}$
$= 2 \pm \sqrt{5}$

Answer: $x = 2 \pm \sqrt{5}$

10. Solve $3x^2 + 2x - 2 = 0$, giving answers to 2 d.p. **[4 marks]**

disc = 4 + 24 = 28
$x = \frac{-2 \pm \sqrt{28}}{6}$
= 0.548... or −1.215...

Answer: x = 0.55 or −1.22

11. Write $x^2 + 6x$ in the form $(x + a)^2 - b$. **[2 marks]**

half of 6 is 3
$(x + 3)^2 = x^2 + 6x + 9$
so subtract the 9 back

Answer: $(x + 3)^2 - 9$

12. Write $x^2 + 8x + 3$ in the form $(x + a)^2 + b$. **[3 marks]**

half of 8 is 4
$(x + 4)^2 - 16 + 3$
$= (x + 4)^2 - 13$

Answer: $(x + 4)^2 - 13$

13. Write $x^2 - 10x + 2$ in completed square form. **[3 marks]**

half of −10 is −5
$(x - 5)^2 - 25 + 2$

Answer: $(x - 5)^2 - 23$

14. Write $x^2 - 3x + 1$ in completed square form. **[3 marks]**

half of −3 is −1.5
$(x - 1.5)^2 - 2.25 + 1$

Answer: $(x - 1.5)^2 - 1.25$

15. Write $2x^2 + 12x + 5$ in the form $a(x + b)^2 + c$. **[4 marks]**

take the 2 out: $2(x^2 + 6x) + 5$
$2[(x + 3)^2 - 9] + 5$
$2(x + 3)^2 - 18 + 5$
the −9 gets multiplied by 2

Answer: $2(x + 3)^2 - 13$

16. The curve $y = (x - 4)^2 + 3$ has a minimum. Write down its coordinates. **[2 marks]**

minimum when the bracket is 0
x = +4, the sign flips

Answer: (4, 3)

17. Write down the minimum value of $(x+2)^2 - 7$ and the x value where it occurs. **[2 marks]**

the square is smallest at 0

Answer: -7 at $x = -2$

18. Write $x^2 + 4x + 9$ in completed square form and hence explain why it is always positive. **[4 marks]**

$(x + 2)^2 - 4 + 9$
$= (x + 2)^2 + 5$
a square is never negative, so the least value is 5

Answer: $(x + 2)^2 + 5$, always ≥ 5

19. Solve $x^2 + 6x + 4 = 0$ by completing the square, giving your answers in surd form. **[4 marks]**

$(x + 3)^2 - 9 + 4 = 0$
$(x + 3)^2 = 5$
$x + 3 = \pm\sqrt{5}$
$x = -3 \pm \sqrt{5}$

Answer: $x = -3 \pm \sqrt{5}$

20. A curve has equation $y = x^2 - 8x + 11$.

(a) Write it in completed square form. **[3 marks]**

half of -8 is -4
$(x - 4)^2 - 16 + 11$
$= (x - 4)^2 - 5$

(b) Write down the coordinates of the turning point. **[2 marks]**

minimum at $x = 4$, $y = -5$
$(4, -5)$

Answer: $(x - 4)^2 - 5$; $(4, -5)$

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