Iteration and Bounds

GCSE Mathematics • **Higher Tier** • Grades 8–9 • Non-calculator

WORKED SOLUTIONS

Name: worked example Class: _____ Total: 69 / 69

Show all your working. Marks are given for correct method as well as the final answer.

1. A length is 7 cm to the nearest cm. Write down its lower and upper bounds. **[2 marks]**

half of 1 cm is 0.5
7 ± 0.5

Answer: 6.5 cm and 7.5 cm

2. A mass is 24 kg to the nearest kg. Write down the error interval. **[2 marks]**

note the $\leq$ and the $<$

Answer: $23.5 \leq m < 24.5$

3. A length is 8.4 cm to 1 d.p. Write down the error interval. **[2 marks]**

half of 0.1 is 0.05

Answer: $8.35 \leq l < 8.45$

4. A value is 250 to 2 significant figures. Write down the error interval. **[3 marks]**

2 s.f. means to the nearest 10
250 ± 5

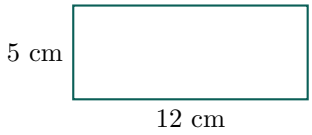
Answer: $245 \leq v < 255$

5. A number is 6.2 when truncated to 1 d.p. Write down the error interval. **[3 marks]**

truncating chops, it does not round
6.2999 truncates to 6.2

Answer: $6.2 \leq n < 6.3$

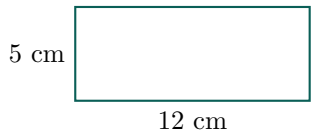
6. A rectangle is 12 cm by 5 cm, each to the nearest cm. Work out the upper bound of its area. **[3 marks]**



upper bounds: 12.5 and 5.5
$12.5 \times 5.5 = 68.75$

Answer: 68.75 **cm^2**

7. Using the same rectangle, work out the lower bound of its area. **[3 marks]**



lower bounds: 11.5 and 4.5
11.5 × 4.5 = 51.75

Answer: 51.75 **cm²**

8. $a = 20$ and $b = 5$, each to the nearest whole number. Work out the upper bound of $a - b$. **[3 marks]**

biggest a = 20.5, SMALLEST b = 4.5
subtracting wants the small one
20.5 − 4.5 = 16

Answer: 16

9. With the same a and b, work out the lower bound of $a - b$. **[3 marks]**

smallest a = 19.5, biggest b = 5.5
19.5 − 5.5 = 14

Answer: 14

10. $a = 30$ and $b = 6$, each to the nearest whole number. Work out the upper bound of $\frac{a}{b}$. **[3 marks]**

biggest top 30.5, SMALLEST bottom 5.5
dividing by less gives more
30.5 ÷ 5.5 = 5.5454...

Answer: 5.545 (3 d.p.)

11. With the same a and b, work out the lower bound of $\frac{a}{b}$. **[3 marks]**

smallest top 29.5, biggest bottom 6.5
29.5 ÷ 6.5 = 4.5384...

Answer: 4.538 (3 d.p.)

12. A car travels 150 m (to the nearest 10 m) in 12 s (to the nearest second). Work out the upper bound of its speed, to 2 d.p. **[4 marks]**

distance up: 155
time DOWN: 11.5
155 ÷ 11.5 = 13.478...

Answer: 13.48 **m/s**

13. Show that $x^3 + x - 3 = 0$ has a root between 1 and 2. **[3 marks]**

x=1: 1 + 1 − 3 = −1
x=2: 8 + 2 − 3 = 7
−1 to +7 is a sign change
so the curve crosses zero between them

Answer: sign change

14. Show that $x^3 - 2x - 5 = 0$ has a root between 2 and 3. **[3 marks]**

$x=2$: $8 - 4 - 5 = -1$
$x=3$: $27 - 6 - 5 = 16$
sign change

Answer: sign change

15. Explain why a sign change shows a root lies between two values. **[2 marks]**

it goes from negative to positive
so it passes through 0 on the way

Answer: the curve must cross zero

16. Use $x_{n+1} = \sqrt{3 - x_n}$ with $x_0 = 1$ to work out x_1, to 3 d.p. **[2 marks]**

$\sqrt{3-1} = \sqrt{2}$
$= 1.41421...$

Answer: 1.414

17. Continue that iteration to work out x_2, to 3 d.p. **[3 marks]**

$\sqrt{3 - 1.41421}$
$= \sqrt{1.58579} = 1.2593...$

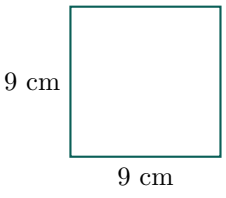
Answer: 1.259

18. Continue to work out x_3, to 3 d.p. **[3 marks]**

$\sqrt{3 - 1.25932}$
$= 1.31935...$

Answer: 1.319

19. A square has side 9 cm to the nearest cm.



(a) Write down the error interval for the side. **[2 marks]**

$8.5 \leq s < 9.5$

(b) Work out the upper and lower bounds of the area. **[3 marks]**

upper: $9.5^2 = 90.25$ cm^2
lower: $8.5^2 = 72.25$ cm^2

Answer: $8.5 \leq s < 9.5$; 72.25 to 90.25 cm^2

20. $x^3 - 6x + 2 = 0$ has a root near $x = 0.3$.

(a) Show the root lies between 0 and 1. **[3 marks]**

$x=0$: $0 - 0 + 2 = 2$

$x=1$: $1 - 6 + 2 = -3$

sign change

(b) Use $x_{n+1} = \dfrac{x_n^3 + 2}{6}$ with $x_0 = 0$ to find x_1 and x_2, to 4 d.p. **[4 marks]**

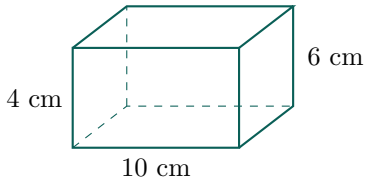
$x_1 = \frac{0+2}{6} = 0.3333$

$x_2 = \frac{0.3333^3+2}{6}$

$= \frac{2.037}{6} = 0.3395$

Answer: sign change ; 0.3333 and 0.3395

21. A cuboid measures 10 cm by 6 cm by 4 cm, each to the nearest cm.



(a) Write down the upper bound of the length. **[1 mark]**

10 ± 0.5

UB = 10.5 cm

(b) Work out the upper bound of the volume. **[3 marks]**

$10.5 \times 6.5 \times 4.5$

all three at their UPPER bound

$= 307.125$ cm^3

Answer: 10.5 cm ; 307.125 cm^3

22. $d = 42.6$ cm and $t = 5.2$ s, both correct to 1 d.p. Work out the upper bound of $d \div t$, to 3 s.f. **[3 marks]**

d UB = 42.65, t LB = 5.15

biggest answer = biggest top, smallest bottom

$42.65 \div 5.15 = 8.281...$

$= 8.28$

Answer: 8.22

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