

Expanding and Factorising Quadratics

GCSE Mathematics • Higher Tier • Grades 4–7 • Non-calculator

WORKED SOLUTIONS

Name: _____ *worked example* _____

Class: _____

Total: _____ *66* _____ / 66

Show all your working. Marks are given for correct method as well as the final answer.

1. Expand and simplify $(x + 4)(x + 6)$. [2 marks]

$$\begin{aligned}x \times x &= x^2 \\x \times 6 &= 6x \\4 \times x &= 4x \\4 \times 6 &= 24 \\6x + 4x &= 10x\end{aligned}$$

Answer: _____ $x^2 + 10x + 24$ _____

2. Expand and simplify $(2y - 5)(y + 7)$. [2 marks]

$$\begin{aligned}2y \times y &= 2y^2 \\2y \times 7 &= 14y \\-5 \times y &= -5y \\-5 \times 7 &= -35\end{aligned}$$

keep the minus with the 5!

Answer: _____ $2y^2 + 9y - 35$ _____

3. Expand and simplify $(x - 3)^2$. [2 marks]

$$\begin{aligned}(x - 3)(x - 3) \\x^2 - 3x - 3x + 9\end{aligned}$$

NOT $x^2 - 9$, you need the middle term

Answer: _____ $x^2 - 6x + 9$ _____

4. Expand and simplify $(3x + 2)^2$. [3 marks]

$$\begin{aligned}(3x + 2)(3x + 2) \\9x^2 + 6x + 6x + 4\end{aligned}$$

Answer: _____ $9x^2 + 12x + 4$ _____

5. Factorise fully $x^2 - 49$. [1 mark]

$$\begin{aligned}x^2 \text{ and } 7^2 \\ \text{difference of two squares}\end{aligned}$$

Answer: _____ $(x + 7)(x - 7)$ _____

6. Factorise fully $9x^2 - 25$. [2 marks]

$$9x^2 = (3x)^2, 25 = 5^2$$

Answer: _____ $(3x + 5)(3x - 5)$ _____

7. Factorise fully $4x^2 - 81$. [2 marks]

$$(2x)^2 - 9^2$$

Answer: _____ $(2x + 9)(2x - 9)$ _____

8. Factorise $x^2 - 3x - 28$. [2 marks]

$\text{need } \times = -28, + = -3$
 $1 \times 28 \quad 2 \times 14 \quad \underline{4 \times 7}$
 $-7 + 4 = -3$

Answer: $\underline{\hspace{1cm}} (x - 7)(x + 4) \underline{\hspace{1cm}}$

9. Factorise $x^2 - 11x + 24$.

[2 marks]

$\times = +24, + = -11 \rightarrow \text{both negative}$
 $\underline{-3 \text{ and } -8}$

Answer: $\underline{\hspace{1cm}} (x - 3)(x - 8) \underline{\hspace{1cm}}$

10. Factorise fully $2x^2 + 10x$.

[2 marks]

$\text{common factor } 2x$
 $\text{fully means take out BOTH}$

Answer: $\underline{\hspace{1cm}} 2x(x + 5) \underline{\hspace{1cm}}$

11. Factorise fully $3x^2 - 27$.

[3 marks]

$\text{take out 3 first: } 3(x^2 - 9)$
 then DOTS inside
 $\text{always take the common factor out first}$

Answer: $\underline{\hspace{1cm}} 3(x + 3)(x - 3) \underline{\hspace{1cm}}$

12. Solve $x^2 - 5x - 14 = 0$.

[3 marks]

$-14: -1 \times 14 \quad -2 \times 7 \quad \underline{-7 \times 2}$
 $(x - 7)(x + 2) = 0$
 $\text{signs flip when you solve!}$
 $x - 7 = 0 \Rightarrow x = 7$
 $x + 2 = 0 \Rightarrow x = -2$

Answer: $\underline{\hspace{1cm}} x = 7 \text{ or } x = -2 \underline{\hspace{1cm}}$

13. Factorise $6x^2 + 11x - 10$.

[3 marks]

$6 \times -10 = -60$
 $\text{need } + = 11 \rightarrow \underline{+15 \text{ and } -4}$
 $6x^2 + 15x - 4x - 10$
 $3x(2x + 5) - 2(2x + 5)$

Answer: $\underline{\hspace{1cm}} (2x + 5)(3x - 2) \underline{\hspace{1cm}}$

14. Solve $2x^2 + 7x + 3 = 0$.

[3 marks]

$2 \times 3 = 6, \text{ need } + = 7 \rightarrow +6 \text{ and } +1$
 $2x^2 + 6x + x + 3$
 $2x(x + 3) + 1(x + 3) = (x + 3)(2x + 1)$
 $x = -3 \text{ or } x = -\frac{1}{2}$

Answer: $\underline{\hspace{1cm}} x = -3 \text{ or } x = -\frac{1}{2} \underline{\hspace{1cm}}$

15. Solve $3x^2 - 10x + 8 = 0$.

[4 marks]

$3 \times 8 = 24, \text{ need } + = -10 \rightarrow -4 \text{ and } -6$
 $3x^2 - 4x - 6x + 8$
 $x(3x - 4) - 2(3x - 4) = (3x - 4)(x - 2)$
 $x = \frac{4}{3} \text{ or } x = 2$

Answer: $x = 2$ or $x = \frac{4}{3}$

16. Expand and simplify $(x + 5)(x - 5)(x + 1)$. [3 marks]

do the DOTS pair first: $(x + 5)(x - 5) = x^2 - 25$
 $(x^2 - 25)(x + 1)$
 $= x^3 + x^2 - 25x - 25$

Answer: $x^3 + x^2 - 25x - 25$

17. Simplify fully $\frac{x^2 - 9}{x^2 + 7x + 12}$. [3 marks]

top: $(x + 3)(x - 3)$ DOTS
bottom: $(x + 3)(x + 4)$
cancel the $(x + 3)$

Answer: $\frac{x-3}{x+4}$

18. Solve $x^2 = 9x$. [3 marks]

$x^2 - 9x = 0$
 $x(x - 9) = 0$
do NOT divide by x , you lose the root $x = 0$

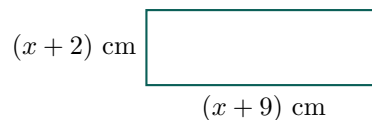
Answer: $x = 0$ or $x = 9$

19. $(x + a)(x + b) = x^2 + 9x + 18$, where a and b are positive integers and $a > b$. Find a and b . [3 marks]

need $\times = 18$, $+= 9$
 1×18 2×9 3×6
 $a > b$ so $a = 6$, $b = 3$

Answer: $a = 6$, $b = 3$

20. A rectangle has width $(x + 2)$ cm and length $(x + 9)$ cm. Its area is 60 cm^2 .



- (a) Show that $x^2 + 11x - 42 = 0$. [2 marks]

area = $(x + 2)(x + 9) = 60$
 $x^2 + 11x + 18 = 60$
 $x^2 + 11x - 42 = 0 \checkmark$

- (b) Find the width, explaining why you reject one solution. [4 marks]

-42 : $+14$ and -3
 $(x + 14)(x - 3) = 0$
 $x = -14$ or $x = 3$
 $x = -14$ gives width -12
no negative lengths, reject!
width = $3 + 2 = 5 \text{ cm}$

Answer: 5 cm

21. Two consecutive positive integers multiply to give 156.

(a) Letting the smaller be n , show that $n^2 + n - 156 = 0$.

[2 marks]

consecutive $\rightarrow n$ and $(n + 1)$

$$n(n + 1) = 156$$

$$n^2 + n - 156 = 0 \checkmark$$

(b) Find the two integers.

[4 marks]

-156: +13 and -12

$$(n + 13)(n - 12) = 0$$

$$n = -13 \text{ or } n = 12$$

must be positive, reject -13

so 12 and 13

Answer: _____ *12 and 13* _____

22. A triangle has base $(x + 3)$ cm and height $(x - 1)$ cm. Its area is 30 cm^2 .

(a) Show that $x^2 + 2x - 63 = 0$.

[3 marks]

$$\text{area} = \frac{1}{2}(x + 3)(x - 1) = 30$$

$$(x + 3)(x - 1) = 60$$

$$x^2 + 2x - 3 = 60$$

$$x^2 + 2x - 63 = 0 \checkmark$$

(b) Find the base of the triangle.

[3 marks]

-63: +9 and -7

$$(x + 9)(x - 7) = 0$$

$$x = -9 \text{ rejected (negative height)}$$

$$x = 7, \text{ base} = 7 + 3 = 10 \text{ cm}$$

Answer: _____ *10 cm* _____