

Expanding and Factorising Quadratics

GCSE Mathematics • Foundation Tier • Grades 3–5 • Non-calculator

WORKED SOLUTIONS

Name: worked example

Class:

Total: 47 / 47

Show all your working. Marks are given for correct method as well as the final answer.

1. Expand $3(x + 5)$. [2 marks]

$$\begin{aligned} 3 \times x &= 3x \\ 3 \times 5 &= 15 \\ \text{multiply BOTH terms} \end{aligned}$$

Answer: $3x + 15$

2. Expand $x(x - 4)$. [2 marks]

$$\begin{aligned} x \times x &= x^2 \\ x \times -4 &= -4x \end{aligned}$$

Answer: $x^2 - 4x$

3. Expand $2x(3x + 7)$. [2 marks]

$$\begin{aligned} 2x \times 3x &= 6x^2 \\ 2x \times 7 &= 14x \end{aligned}$$

Answer: $6x^2 + 14x$

4. Expand and simplify $(x + 2)(x + 5)$. [2 marks]

$$\begin{aligned} x \times x &= x^2 \\ x \times 5 &= 5x \\ 2 \times x &= 2x \\ 2 \times 5 &= 10 \\ 5x + 2x &= 7x \end{aligned}$$

Answer: $x^2 + 7x + 10$

5. Expand and simplify $(x + 7)(x - 3)$. [2 marks]

$$\begin{aligned} x \times x &= x^2 \\ x \times -3 &= -3x \\ 7 \times x &= 7x \\ 7 \times -3 &= -21 \\ -3x + 7x &= +4x \end{aligned}$$

Answer: $x^2 + 4x - 21$

6. Expand and simplify $(x - 4)(x - 6)$. [2 marks]

$$\begin{aligned} x \times -6 &= -6x \\ -4 \times x &= -4x \\ -4 \times -6 &= +24 \\ \text{minus} \times \text{minus} &= \text{plus!} \end{aligned}$$

Answer: $x^2 - 10x + 24$

7. Expand and simplify $(2x + 1)(x + 4)$. [2 marks]

$$\begin{aligned} 2x \times x &= 2x^2 \\ 2x \times 4 &= 8x \end{aligned}$$

$$1 \times x = x$$

$$1 \times 4 = 4$$

$$8x + x = 9x$$

Answer: $2x^2 + 9x + 4$

8. Expand and simplify $(x + 5)^2$. [2 marks]

$$(x + 5)(x + 5)$$

$$x^2 + 5x + 5x + 25$$

NOT $x^2 + 25$ -- you need the middle term

Answer: $x^2 + 10x + 25$

9. Expand and simplify $(x - 3)^2$. [2 marks]

$$(x - 3)(x - 3)$$

$$x^2 - 3x - 3x + 9$$

Answer: $x^2 - 6x + 9$

10. Expand and simplify $(x + 4)(x - 4)$. What do you notice? [2 marks]

$$x^2 - 4x + 4x - 16$$

the x terms cancel

this is the difference of two squares

Answer: $x^2 - 16$

11. Factorise $x^2 + 7x$. [2 marks]

both terms have an x
take x outside

Answer: $x(x + 7)$

12. Factorise fully $6x^2 - 9x$. [2 marks]

common number: 3
common letter: x
fully means take out BOTH

Answer: $3x(2x - 3)$

13. Factorise $x^2 + 8x + 15$. [2 marks]

need $\times = 15$, $+$ = 8

$$1 \times 15 \quad 3 \times 5 \leftarrow 3 + 5 = 8$$

Answer: $(x + 3)(x + 5)$

14. Factorise $x^2 - 9x + 20$. [2 marks]

$\times = +20$, $+$ = $-9 \rightarrow$ *both negative*
-4 and -5

Answer: $(x - 4)(x - 5)$

15. Factorise $x^2 + 2x - 15$. [2 marks]

$\times = -15$, $+$ = $+2$
-3 and +5
one sign must be minus

Answer: $(x - 3)(x + 5)$

16. Factorise $x^2 - 3x - 28$.

[2 marks]

$$\begin{aligned} \times &= -28, + = -3 \\ 1 \times 28 & \quad 2 \times 14 & \quad \underline{4 \times 7} \\ -7 + 4 &= -3 \end{aligned}$$

Answer: $(x - 7)(x + 4)$

17. Factorise $x^2 - 36$.

[2 marks]

$$\begin{aligned} x^2 \text{ and } 36 &= 6^2 \\ \text{difference of two squares} \end{aligned}$$

Answer: $(x + 6)(x - 6)$

18. Solve $(x - 2)(x + 9) = 0$.

[2 marks]

$$\begin{aligned} x - 2 &= 0 \Rightarrow x = 2 \\ x + 9 &= 0 \Rightarrow x = -9 \\ \text{the signs flip!} \end{aligned}$$

Answer: $x = 2 \text{ or } x = -9$

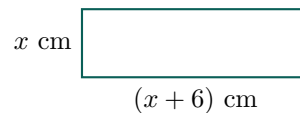
19. Solve $x^2 + 5x + 6 = 0$ by factorising.

[3 marks]

$$\begin{aligned} \times &= 6, + = 5 \rightarrow \underline{2 \text{ and } 3} \\ (x + 2)(x + 3) &= 0 \\ x &= -2 \text{ or } x = -3 \end{aligned}$$

Answer: $x = -2 \text{ or } x = -3$

20. Here is a rectangle. Its area is 27 cm^2 .



(a) Show that the area is $x^2 + 6x$.

[2 marks]

$$\begin{aligned} \text{area} &= w \times l = x(x + 6) \\ &= x^2 + 6x \checkmark \end{aligned}$$

(b) Show that $x^2 + 6x - 27 = 0$, then factorise the left-hand side.

[3 marks]

$$\begin{aligned} x^2 + 6x &= 27 \\ x^2 + 6x - 27 &= 0 \checkmark \\ -27: +9 \text{ and } -3 &\leftarrow 9 - 3 = 6 \\ (x + 9)(x - 3) \end{aligned}$$

(c) Find the value of x , explaining why you reject one solution.

[3 marks]

$$\begin{aligned} (x + 9)(x - 3) &= 0 \\ x &= -9 \text{ or } x = 3 \\ x = -9 &\text{ gives a negative length} \\ \text{reject it} \\ \text{so } x &= 3 \end{aligned}$$

Answer: $x = 3 \text{ cm}$

