Composite and Inverse Functions

GCSE Mathematics • **Higher Tier** • Grades 7–9 • Non-calculator

WORKED SOLUTIONS

Name: worked example Class: _____ Total: 58 / 58

Show all your working. Marks are given for correct method as well as the final answer.

1. $f(x) = 3x - 5$. Work out $f(4)$. **[1 mark]**

$3(4) - 5$
$= 12 - 5 = 7$

Answer: 7

2. $f(x) = 3x - 5$. Work out $f(-2)$. **[2 marks]**

$3(-2) - 5$
$= -6 - 5 = -11$

Answer: -11

3. $g(x) = x^2 + 1$. Work out $g(-3)$. **[2 marks]**

$(-3)^2 + 1$
squaring kills the minus
$= 9 + 1 = 10$

Answer: 10

4. $f(x) = 2x + 7$. Solve $f(x) = 19$. **[2 marks]**

$2x + 7 = 19$
$2x = 12$
$x = 6$

Answer: $x = 6$

5. $g(x) = x^2 - 4$. Solve $g(x) = 21$. **[3 marks]**

$x^2 - 4 = 21$
$x^2 = 25$
a square root gives TWO answers
$x = \pm 5$

Answer: $x = 5$ or $x = -5$

6. $f(x) = 2x$ and $g(x) = x + 3$. Work out $fg(4)$. **[2 marks]**

fg means do g FIRST
work from the inside out
$g(4) = 7$
$f(7) = 14$

Answer: 14

7. $f(x) = 2x$ and $g(x) = x + 3$. Work out $gf(4)$. **[2 marks]**

$f(4) = 8$
$g(8) = 11$
different from $fg(4) = 14$

Answer: 11

8. $f(x) = 2x$ and $g(x) = x + 3$. Work out $fg(x)$. **[2 marks]**

$g(x) = x + 3$
$f(x + 3) = 2(x + 3)$
$= 2x + 6$

Answer: $2x + 6$

9. $f(x) = 2x$ and $g(x) = x + 3$. Work out $gf(x)$. **[2 marks]**

$f(x) = 2x$
$g(2x) = 2x + 3$

Answer: $2x + 3$

10. $f(x) = x^2$ and $g(x) = x - 1$. Work out $fg(x)$, expanded. **[3 marks]**

$g(x) = x - 1$
$f(x - 1) = (x - 1)^2$
$= x^2 - 2x + 1$

Answer: $x^2 - 2x + 1$

11. $f(x) = x^2$ and $g(x) = x - 1$. Work out $gf(x)$. **[2 marks]**

$f(x) = x^2$
$g(x^2) = x^2 - 1$

Answer: $x^2 - 1$

12. $f(x) = 3x + 1$. Work out $ff(x)$. **[3 marks]**

$f(3x + 1) = 3(3x + 1) + 1$
$= 9x + 3 + 1$
$= 9x + 4$

Answer: $9x + 4$

13. $f(x) = x + 8$. Work out $f^{-1}(x)$. **[2 marks]**

the inverse undoes the function
adding 8 undoes to −8

Answer: $x - 8$

14. $f(x) = 4x$. Work out $f^{-1}(x)$. **[2 marks]**

$\times 4$ undoes to $\div 4$

Answer: $\frac{x}{4}$

15. $f(x) = 5x - 3$. Work out $f^{-1}(x)$. **[3 marks]**

write $y = 5x - 3$
$y + 3 = 5x$
$x = \frac{y+3}{5}$
swap the letter back to x at the end

Answer: $\frac{x+3}{5}$

16. $f(x) = \frac{x}{2} + 6$. Work out $f^{-1}(x)$. **[3 marks]**

$y = \frac{x}{2} + 6$
$y - 6 = \frac{x}{2}$
$x = 2(y - 6)$

Answer: $2(x - 6)$

17. $f(x) = 3x + 2$. Work out $f^{-1}(11)$. **[3 marks]**

$f^{-1}(x) = \frac{x-2}{3}$
$\frac{11-2}{3} = 3$
check: $f(3) = 11$ ✓

Answer: 3

18. $f(x) = 4x - 1$ and $g(x) = x^2$.

(a) Work out $fg(3)$. **[2 marks]**

$g(3) = 9$
$f(9) = 36 - 1 = 35$

(b) Work out $gf(3)$. **[2 marks]**

$f(3) = 11$
$g(11) = 121$

Answer: 35 ; 121

19. $f(x) = 2x - 7$.

(a) Work out $f^{-1}(x)$. **[3 marks]**

$y = 2x - 7$
$y + 7 = 2x$
$f^{-1}(x) = \frac{x+7}{2}$

(b) Solve $f(x) = f^{-1}(x)$. **[4 marks]**

$2x - 7 = \frac{x+7}{2}$
$4x - 14 = x + 7$
$3x = 21$
$x = 7$

Answer: $\frac{x+7}{2}$; $x = 7$

20. $f(x) = x + 4$ and $g(x) = 3x$.

(a) Show that $fg(x) \neq gf(x)$. **[4 marks]**

$fg(x) = f(3x) = 3x + 4$

$gf(x) = g(x + 4) = 3(x + 4) = 3x + 12$

$3x + 4 \neq 3x + 12$ ✓

(b) Solve $fg(x) = gf(x) + 8$. **[4 marks]**

$3x + 4 = 3x + 12 + 8$

$3x + 4 = 3x + 20$

$4 = 20$ is impossible

no solution -- the x terms cancel

Answer: ____ $3x + 4$ vs $3x + 12$; no solution ____

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