# Circle Theorems

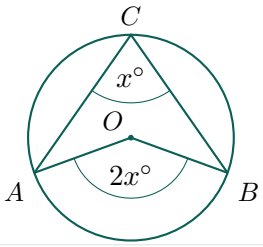
GCSE Mathematics • **Higher Tier** • Grades 8–9 • Non-calculator

**WORKED SOLUTIONS**

Name: worked example Class: _____ Total: 55 / 55

*Show all your working. Marks are given for correct method as well as the final answer.*

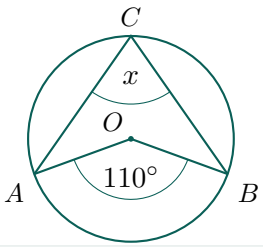
**1.** The diagram shows an angle at the centre and an angle at the circumference on the same arc. State the theorem that connects them. **[2 marks]**




the angle at the centre is twice the angle at the circumference on the same arc

**Answer:** centre = 2 × circumference

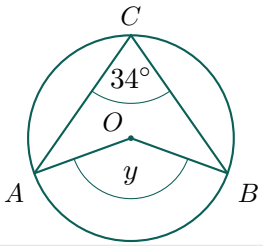
**2.** Find $x$, giving a reason. **[3 marks]**




110 ÷ 2 = 55
angle at centre = 2 × circumference

**Answer:** x = 55°

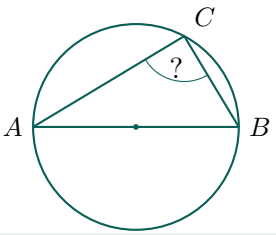
**3.** Find $y$, the angle at the centre. **[2 marks]**




34 × 2 = 68
doubling this time, not halving

**Answer:** 68°

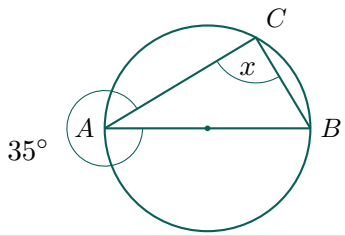
**4.** $AB$ is a diameter. Write down the size of angle $ACB$, and name the theorem. **[2 marks]**




an angle in a semicircle is a right angle

**Answer:** 90°

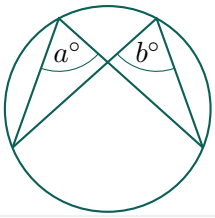
**5.** $AB$ is a diameter. Work out angle $x$. **[3 marks]**




angle at C = 90° (semicircle)
180 – 90 – 35 = 55°

**Answer:** 55°

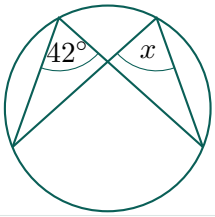
**6.** The two marked angles stand on the same arc. State the theorem that connects them. **[2 marks]**




angles in the same segment are equal

**Answer:** they are equal

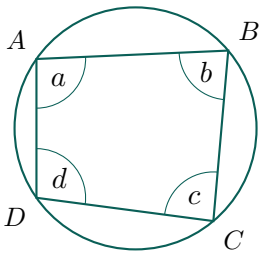
**7.** Write down the size of $x$, giving a reason. **[2 marks]**




angles in the same segment are equal

**Answer:** 42°

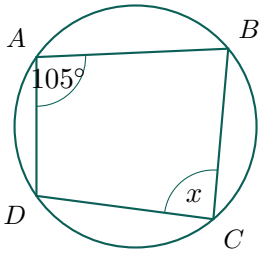
**8.** All four vertices lie on the circle. State the theorem about its opposite angles. **[2 marks]**




a + c = 180° and b + d = 180°

**Answer:** opposite angles add to 180°

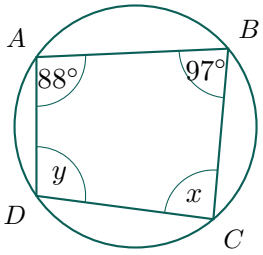
**9.** Work out $x$. **[2 marks]**




opposite angles add to 180
180 – 105 = 75

**Answer:** 75°

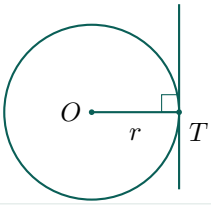
**10.** Work out $x$ and $y$. **[3 marks]**




x is opposite 88: 180 – 88 = 92
y is opposite 97: 180 – 97 = 83

**Answer:** x = 92°, y = 83°

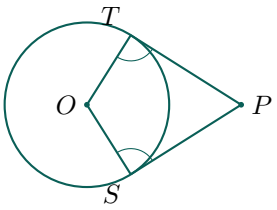
**11.** $OT$ is a radius and the line through $T$ is a tangent. Write down the angle at $T$, with a reason. **[2 marks]**




a tangent meets a radius at 90°

**Answer:** 90°

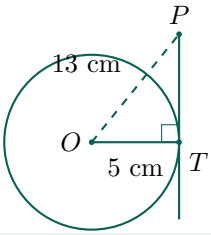
**12.** Two tangents are drawn to the circle from the point $P$. What can you say about their lengths? **[2 marks]**




tangents from a point are equal in length

**Answer:** they are equal (PT = PS)

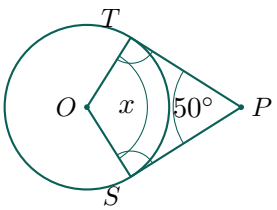
**13.** $OT$ is a radius and $PT$ is a tangent. Work out the length $PT$. **[4 marks]**




the radius meets the tangent at 90°
so use Pythagoras
169 – 25 = 144
$\sqrt{144}$ = 12

**Answer:** 12 **cm**

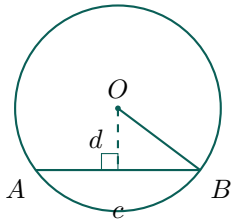
**14.** Two tangents meet at $P$, where angle $P = 50°$. Work out the angle at the centre between the two radii. **[4 marks]**




OTPS is a quadrilateral
each radius meets its tangent at 90°
angles in a quadrilateral add to 360°
360 – 90 – 90 – 50 = 130°

**Answer:** 130°

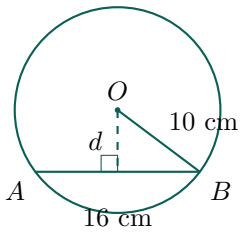
**15.** $O$ is the centre and the dashed line is perpendicular to the chord. State what it does to the chord. **[2 marks]**




*a perpendicular from the centre splits a chord into two equal halves*

**Answer:** *it bisects the chord*

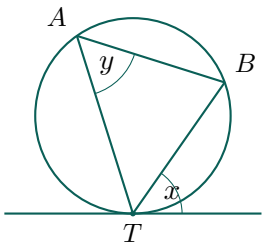
**16.** Work out $d$, the distance of the chord from the centre. **[4 marks]**




*the perpendicular bisects: half-chord = 8*
*radius 10 is the hypotenuse*
*100 – 64 = 36*
*d = 6 cm*

**Answer:** *6* **cm**

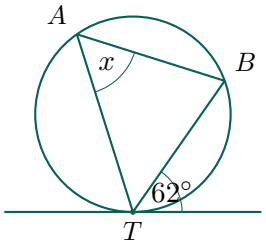
**17.** State the alternate segment theorem, using the angles marked on the diagram. **[2 marks]**




*the angle between a tangent and a chord equals the angle in the alternate segment*

**Answer:** *x = y*

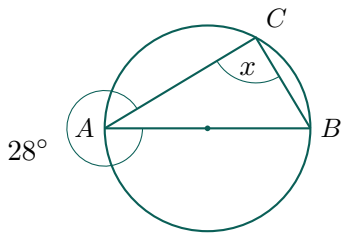
**18.** Work out $x$. **[2 marks]**




*alternate segment theorem*
*so x = 62°*

**Answer:** *62°*

**19.** $AB$ is a diameter and angle $CAB = 28°$.




**(a)** Write down angle $ACB$, with a reason. **[2 marks]**

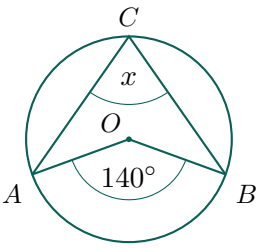
90°
angle in a semicircle

**(b)** Work out angle $x$. **[3 marks]**

angles in a triangle add to 180
180 − 90 − 28
= 62°

**Answer:** 90° ; 62°

**20.** $O$ is the centre and the angle at the centre is 140°.




**(a)** Work out $x$, the angle at the circumference. **[2 marks]**

140 ÷ 2
= 70°

**(b)** A cyclic quadrilateral is drawn using that angle. Work out the angle opposite it. **[3 marks]**

opposite angles add to 180
180 − 70
= 110°

**Answer:** 70° ; 110°

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