Algebraic Proof

GCSE Mathematics • **Higher Tier** • Grades 8–9 • Non-calculator

WORKED SOLUTIONS

Name: worked example Class: _____ Total: 63 / 63

Show all your working. Marks are given for correct method as well as the final answer.

1. Write down an expression for an even number. **[1 mark]**

n is any integer
2n is always even

Answer: 2n

2. Write down an expression for an odd number. **[1 mark]**

one more than an even number
2n – 1 is also fine

Answer: 2n + 1

3. Write down expressions for three consecutive integers. **[2 marks]**

each is one more than the last

Answer: n, n + 1, n + 2

4. Write down expressions for two consecutive EVEN numbers. **[2 marks]**

consecutive evens differ by 2
NOT 2n and 2n + 1

Answer: 2n and 2n + 2

5. Prove that the sum of two even numbers is always even. **[3 marks]**

let them be 2m and 2n
2m + 2n = 2(m + n)
m + n is an integer, so this is 2 × an integer
so the sum is always even

Answer: 2m + 2n = 2(m + n)

6. Prove that the sum of two odd numbers is always even. **[3 marks]**

(2m + 1) + (2n + 1)
= 2m + 2n + 2
= 2(m + n + 1)
a multiple of 2, so even

Answer: 2(m + n + 1)

7. Prove that the product of two odd numbers is always odd. **[4 marks]**

(2m + 1)(2n + 1)
= 4mn + 2m + 2n + 1
= 2(2mn + m + n) + 1
one more than an even number, so odd

Answer: 2(2mn + m + n) + 1

8. Prove that the sum of three consecutive integers is always a multiple of 3. **[3 marks]**

$n + (n + 1) + (n + 2)$
$= 3n + 3$
$= 3(n + 1)$
so it is always a multiple of 3

Answer: $3(n + 1)$

9. Prove that the sum of four consecutive integers is never a multiple of 4. **[4 marks]**

$n + (n+1) + (n+2) + (n+3)$
$= 4n + 6$
$= 4(n + 1) + 2$
always 2 more than a multiple of 4

Answer: $4n + 6 = 2(2n + 3)$, not a multiple of 4

10. Prove that the difference between the squares of two consecutive integers is always odd. **[4 marks]**

$(n + 1)^2 - n^2$
$= n^2 + 2n + 1 - n^2$
$= 2n + 1$
which is odd

Answer: $2n + 1$

11. Prove that $(n + 3)^2 - (n - 3)^2$ is always a multiple of 12. **[4 marks]**

$(n^2 + 6n + 9) - (n^2 - 6n + 9)$
$= 12n$
watch the signs on the second bracket

Answer: $12n$

12. Prove that $(2n + 1)^2 - (2n - 1)^2$ is always a multiple of 8. **[4 marks]**

$(4n^2 + 4n + 1) - (4n^2 - 4n + 1)$
$= 8n$

Answer: $8n$

13. Prove that the sum of the squares of two consecutive integers is always odd. **[4 marks]**

$n^2 + (n + 1)^2$
$= 2n^2 + 2n + 1$
$= 2(n^2 + n) + 1$
so it is odd

Answer: $2(n^2 + n) + 1$

14. Prove that the product of two consecutive integers is always even. **[3 marks]**

$n(n + 1)$
of any two consecutive integers, one is even
an even factor makes the product even

Answer: one of them must be even

15. Kim says n^2 *is always bigger than* n. Give a counter example. **[2 marks]**

$1^2 = 1$, which is not bigger
$n = 0$ also works

Answer: $n = 1$

16. Sam says *every prime number is odd.* Give a counter example. **[1 mark]**

2 is prime and even

Answer: 2

17. Alex says *the square root of a number is always smaller than the number.* Give a counter example. **[2 marks]**

$\sqrt{0.25}$ = 0.5, which is bigger
it breaks for numbers below 1

Answer: 0.25

18. Explain why one counter example is enough to disprove a statement, but examples can never prove one. **[3 marks]**

always means no exceptions
one exception destroys it
but no number of examples covers every case

Answer: a claim must hold for every case

19. n is an integer.

(a) Prove that $(n+1)^2 - (n-1)^2$ is always a multiple of 4. **[4 marks]**

$(n^2 + 2n + 1) - (n^2 - 2n + 1)$
$= 4n$
a multiple of 4

(b) Explain why it is also always even. **[2 marks]**

$4n = 2(2n)$
every multiple of 4 is a multiple of 2

Answer: $4n$; $4n = 2(2n)$

20. n is an integer.

(a) Prove that the sum of any five consecutive integers is a multiple of 5. **[4 marks]**

$n + (n+1) + (n+2) + (n+3) + (n+4)$
$= 5n + 10$
$= 5(n + 2)$
a multiple of 5

(b) Write down the middle number when that sum is 90. **[3 marks]**

$5(n + 2) = 90$
$n + 2 = 18$
$n + 2$ IS the middle number
$= 18$

Answer: $5(n + 2)$; the middle number is 18

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